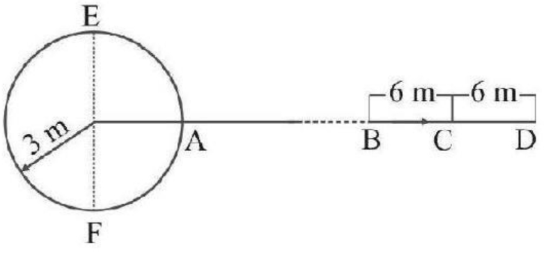
Waves

1. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of $10 \mathrm{~km/h}$. If the wave speed is $330 \mathrm{~m/s}$, the frequency (in Hz) heard by the running person shall be

2. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be: (Assume that the highest frequency a person can hear is $20{,}000 \mathrm{~Hz}$)

3. Equation of travelling wave on a stretched string of linear density $5 \mathrm{~g/m}$ is $\mathrm{y}=0.03 \sin (450 \mathrm{t}-9 \mathrm{x})$ where distance and time are measured in SI units. The tension (in N) in the string is:

4. A source of sound is moving along a circular orbit of radius 3 m with an angular velocity of $10 \mathrm{rad/s}$. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with an amplitude $BC=CD=6$ m. The frequency of oscillation of the detector is $5/\pi$ per second. The source is at the point A when the detector is at the point B. If the source emits a continuous sound wave of frequency 340 Hz, find the minimum frequencies (in Hz) recorded by the detector.



5. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of $20 \mathrm{~ms}^{-1}$ with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency (in Hz) of the sound source in car B? (speed of sound in air $=340 \mathrm{~ms}^{-1}$)

6. The amplitude of a wave disturbance propagating along positive x-axis is given by $y=\frac{1}{(1+x^2)}$ at $t=0$ and $y=\frac{1}{1+(x-2)^2}$ at $t=4$ s, where x and y are in metre. The shape of wave disturbance does not change with time. Find velocity (in $\mathrm{m/s}$) of the wave.

7. A tuning fork of frequency 220 Hz produces sound waves of wavelength 1.5 m in air at STP. Calculate the increase in wavelength, (in metre) when temperature of air is $27^{\circ}\mathrm{C}$.

8. A sample of oxygen at NTP has volume V and a sample of hydrogen at NTP has volume $4V$. Both the gases are mixed and the mixture is maintained at NTP. If the speed of sound in hydrogen at NTP is $1270 \mathrm{~m/s}$, calculate the speed of sound (in $\mathrm{m/s}$) in the mixture.

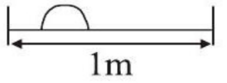
9. A glass tube 1.0 m length is filled with water. The water can be drained out slowly at the bottom of the tube. If a vibrating tuning fork of frequency $500 \mathrm{c/s}$ is brought at the upper end of the tube and the velocity of sound is $330 \mathrm{~m/s}$, then the total number of resonances obtained will be

10. Pipe A has length twice the pipe B. Pipe A has both ends open and pipe B has one end open. Which harmonics of pipe A have a frequency that matches a resonance frequency of pipe B

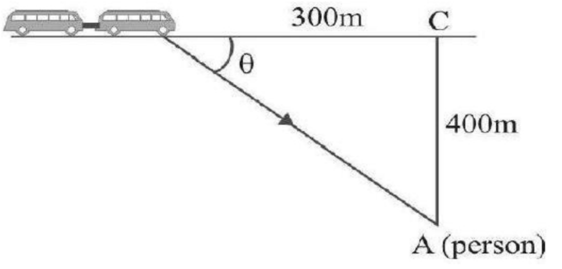
11. A string is clamped at both the ends and it is vibrating in its 4^{th} harmonic. The equation of the stationary wave is $\mathrm{Y}=0.3 \sin (0.157 \mathrm{x}) \cos (200 \pi \mathrm{t})$. The length (in metre) of the string is: (All quantities are in SI units.)

12. A road runs between two parallel rows of buildings. A motorist moving just in the middle with a velocity of $30 \text{ km}/h$, sounds the horn. He hears an echo one second after sounding the horn. Find the distance (in metre) between the two rows of the buildings. The velocity of sound $= 330 \text{ m/s}$.

13. One metre long wire is fixed between two rigid supports. The tension is the wire is 200 N and mass per unit length of the wire is $\mu = 2x$, where x is the distance from one end of the wire. Find the time (in second) the pulse takes to reach the other end of the wire.



14. A train approaching a railway crossing at a speed of 120 km/h sounds a short whistle at frequency 640 Hz when it is 300 m away from the crossing. The speed of sound in air is 340 m/s. What will be the frequency (in Hz) heard by a person standing on a road perpendicular to the track through the crossing at a distance of 400 m from the crossing?



15. A string of length 40 cm and weighing 10 g is attached to a spring at one end and to a fixed wall at the other end. The spring has a spring constant of 160 N/m and is stretched by 1.0 cm. If a wave pulse is produced on the string near the wall, the time it will take to reach the spring is $\frac{1}{x}$. The value of x (in second) is

SOLUTIONS

1. **(666)** Frequency of the sound produced by open flute.

$$f = 2\left(\frac{v}{2\ell}\right) = \frac{2\times330}{2\times0.5} = 660Hz$$

Velocity of observer, $v_0 = 10\times\frac{5}{18} = \frac{25}{9} m/s$

As the source is moving towards the observer therefore, according to Doppler's effect.

$\therefore$ Frequency detected by observer,

$$f' = \left[\frac{v+v_0}{v}\right]f = \left[\frac{\frac{25}{9}+330}{330}\right]660$$

$$= \frac{2995}{9\times330}\times660 \text{ or, } f' = 665.55 \simeq 666 \text{ Hz}$$

2. **(6)** If a closed pipe vibration in N^{th} mode then frequency of vibration $n = \frac{(2N-1)v}{4l} = (2N-1)n_1$

(where n_1 = fundamental frequency of vibration)

Hence $20{,}000 = (2N-1)\times1500$

$\Rightarrow$ $N = 7.1 \approx 7$

$\therefore$ Number of over tones = (No. of mode of vibration) – 1

$= 7 - 1 = 6$

3. **(12.5)** We have given,

$y = 0.03\sin(450t - 9x)$

Comparing it with standard equation of wave, we get

$\omega = 450\ k = 9$

$\therefore\ v = \frac{\omega}{k} = \frac{450}{9} = 50 m/s$

Velocity of travelling wave on a stretched string is given by

$v = \sqrt{\frac{T}{\mu}} \Rightarrow \frac{T}{\mu} = 2500$

μ = linear mass density

$\Rightarrow T = 2500\times5\times10^{-3}$

$\Rightarrow 12.5$ N

4. **(255)** The maximum velocity of the detector

$$v_0 = \omega A$$
$$= 2\pi f A$$
$$= 2\pi\times\frac{5}{\pi}\times6 = 60 \text{ m/s.}$$

The velocity of the source,

$$v_s = \omega r = 10\times3 = 30 \text{ m/s.}$$

If source is going clockwise on the circular orbit, then maximum will be heard at E and minimum frequency at F.

Thus $$f_{min} = f\left(\frac{v-v_0}{v+v_s}\right)$$

$$= 340\left(\frac{340-60}{340+30}\right) \simeq 255 Hz.$$

5. **(2250)** $f' = f\dfrac{v-v_0}{v+v_s}$

or $2000 = f\dfrac{340-20}{340+20}$

$\therefore$ $f = 2250$ Hz.

6. **(0.5)** The equation of the wave at any time t can be obtained by putting $(x - vt)$ in place of x in the given expression, so we have

$$y = \frac{1}{1+(x-vt)^2} \quad \text{...(i)}$$

Given $y = \dfrac{1}{1+(x-2)^2}$ at $t = 4$s ...(ii)

On comparing equations (i) and (ii), we get

$vt = 2$

As $t = 4$s, $\therefore$ $v = \dfrac{2}{t} = \dfrac{2}{4} = 0.5$ m/s.

7. **(0.07)** Given $f = 220$ Hz, $\lambda_0 = 1.5$ m at $T_0 = 273$ K

Speed of sound at STP, $v_0 = f\lambda_0 = 220 \times 1.5 = 330$ m/s.

Final temperature, $T = 273 + 27 = 300$ K

Let v the speed of sound at this temperature, then

$$\frac{v}{v_0} = \sqrt{\frac{T}{T_0}}$$

$$\therefore \quad v = v_0\sqrt{\frac{T}{T_0}}$$

$$= 330\sqrt{\frac{300}{273}} = 346.1 \text{ m/s}$$

Final wavelength, $\lambda = \dfrac{v}{f} = \dfrac{346.1}{220} = 1.57$ m

The increase in wavelength $= \lambda - \lambda_0 = 1.57 - 1.50 = 0.07$ m.

8. **(635)** If V_H and V_m are the velocities in hydrogen and mixture respectively, then

$$\frac{v_m}{v_H} = \sqrt{\frac{\rho_H}{\rho_m}} \quad \text{...(i)}$$

Density of mixture, $\rho_m = \dfrac{\rho_o V_o + \rho_H V_H}{V_o + V_H}$

where ρ_0 and V_0 are the density and volume of the oxygen.

$$\rho_m = \frac{\rho_H V_H \left(1 + \frac{\rho_o}{\rho_H} \times \frac{V_o}{V_H}\right)}{V_H \left(1 + \frac{V_o}{V_H}\right)}$$

or
$$\frac{\rho_m}{\rho_H} = \frac{\left(1 + \frac{\rho_o}{\rho_H} \times \frac{V_o}{V_H}\right)}{\left(1 + \frac{V_o}{V_H}\right)}$$

$$= \frac{1 + 16 \times \frac{1}{4}}{1 + \frac{1}{4}} = 4$$

or
$$\frac{\rho_H}{\rho_m} = \frac{1}{4}$$

From equation (i),

$$v_m = v_H \sqrt{\frac{1}{4}} = \frac{v_H}{2}$$

$$= \frac{1270}{2} = 635 \text{ m/s.}$$

9. **(3)** $\lambda = \dfrac{v}{f} = \dfrac{330}{500} = 0.66 \text{ m}$

The resonance lengths are :

$$\ell_1 = \frac{\lambda}{4} = 0.165 \text{ m},$$

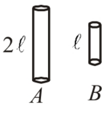
$$\ell_2 = \frac{3\lambda}{4} = 0.495 \text{ m},$$

$$\ell_3 = \frac{5\lambda}{4} = 0.825 \text{ m},$$

and
$$\ell_4 = \frac{7\lambda}{4} = 1.155 \text{ m}$$

As ℓ_4 is greater than 1 m, so allowed resonances are only three.

10. **(1)** $f_A = \dfrac{v}{2(2\ell)}$ and $f_B = \dfrac{v}{4\ell}$



Clearly first harmonic of both the pipes have equal frequency.

11. **(80)** Given, y = 0.3 sin (0.157 x) cos (200 πt)

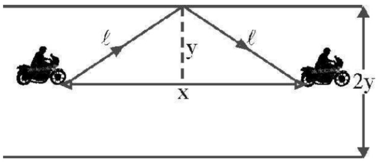
So, k = 0.157 and ω = 200π = 2πf

$\therefore$ f = 100 Hz and $v = \dfrac{\omega}{k} = \dfrac{200\pi}{0.157} = 4000 \text{m/s}$

Now, using $f = \dfrac{nv}{2l} = \dfrac{4v}{2l} = \dfrac{2v}{l}$ [here n = 4]

$\therefore l = \dfrac{2v}{f} = \dfrac{2 \times 4000}{100} = 80m$

12. **(329.9)** Suppose 2 y is the distance between two rows of the buildings. The distance travelled by car in 1 second



$$x = \left(30 \times \frac{5}{18}\right) \times 1 = 8.33\,\text{m}$$

The distance travelled by sound in 1 second

$$2\ell = 330 \times 1 = 330\,\text{m}$$

$\therefore$ $$\ell = 165\,\text{m}$$

From the figure $$y = \sqrt{\ell^2 - \left(\frac{x}{2}\right)^2}$$

$$= \sqrt{165^2 - \left(\frac{8.33}{2}\right)^2}$$

$$= 164.95\,\text{m}$$

Thus the distance between two rows of the buildings $2y = 329.9\,\text{m}$.

13. **(0.067)** The pulse velocity is given by,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{200}{2x}} = \frac{10}{\sqrt{x}}$$

or $$\frac{dx}{dt} = \frac{10}{\sqrt{x}}$$

or $$\int_0^1 \sqrt{x}\,dx = 10\int_0^t dt$$

or $$\int_0^1 x^{1/2}dx = 10\int_0^t dt$$

or $$\left|\frac{x^{3/2}}{3/2}\right|_0^1 = 10\,\text{t}$$

or $$\text{t} = \frac{2}{30}\,\text{s} = 0.067\,\text{s}$$

14. **(680)** From the geometry of the figure

$$\cos\theta = \frac{3}{5}.$$

The frequency heard by the person

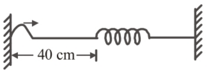
$$f' = f\frac{v}{v - v_s\cos\theta}.$$

Here $$v_s = 120 \times \frac{5}{18} = 33.3\text{ m/s}.$$

$\therefore$ $$f' = 640 \times \frac{340}{340 - 33.3 \times \frac{3}{5}} = 680\text{ Hz}.$$

15. **(20)** The tension in the string

$$F = kx = 160 \times 0.01 = 1.6\,\text{N}$$



The mass per unit length of the string

$$\mu = \frac{10 \times 10^{-3}}{0.40} = 0.025\,\text{kg/m}$$

The speed of transverse wave

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{1.6}{0.025}} = 8\text{ m/s}.$$

The time taken by wave pulse to reach the spring

$$t = \frac{0.40}{8} = 0.05\text{ s}$$